Shell-Model States and Configuration Mixing in the Ti Isotopes by the (*p*,*d*) Reaction*

E. KASHY AND T. W. CONLON
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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The energy spectra and angular distributions of deuterons resulting from the bombardment of Ti^{46}, Ti^{47}, Ti^{48}, Ti^{49}, and Ti^{50} with 17.5-MeV protons have been measured. For even-A targets, one strong transition is observed while for odd-A targets a large number of levels are strongly excited. The deuteron angular distributions show that these strong transitions are due to the pickup of an $l_n=3$ neutron. A number of even-l transitions corresponding to the pickup of neutrons from the already filled $1d_{3/2}$ and $2s_{1/2}$ shells have also been observed. The Q values for the pickup of $l_n=2$ neutrons from the even-A Ti isotopes show an interesting feature in that they are found to be approximately independent of the Ti isotope used. A few $l_n=1$ transitions have also been observed indicating p-wave admixture in the ground level wave functions of the target nuclei. By comparison of the experimental result with DWBA calculation, spectroscopic factors have been calculated for the transitions observed. The spectroscopic factors and excitation energies for $l_n=3$ transitions have been compared to recent theoretical predictions where it is assumed that protons and neutrons in excess of 20 can be treated as belonging to a pure $(1f_{7/2})^n$ configuration.

INTRODUCTION

THE titanium isotopes provide an ideal set of nuclei for studying the systematics of a particular reaction such as the present (*p*,*d*) investigation. There are five stable Ti isotopes, $Z=22$, $N=24$ to 28, so that all protons and neutrons in excess of 20 may be considered to belong mainly to the $(1f_{7/2})$ configuration. The main transitions expected therefore are those corresponding to the removal of a $(1f_{7/2})$ neutron, i.e., $l_n=3$ transitions. Other types of transitions are possible if the ground levels of the target isotopes are admixed with higher shell-model configurations, in particular, the $2p_{3/2}$ configuration, which is the more likely since it lies lowest in excitation energy. Transitions can also occur corresponding to the pickup of particles from shells which are already filled. All these types of transitions have been observed in the present investigation.

The (*p*,*d*) reactions are especially effective in providing information about the ground levels of nuclei. They provide an almost direct measure of the various components of the wave function in terms of the overlap of initial and final states, assuming we have a theory of the reaction mechanism. This is the case in distorted wave Born approximation (DWBA) calculation of (*p*,*d*) cross sections, which uses optical potentials to represent the non-Coulomb interaction of the incident and outgoing particles with the target and daughter nucleus.[1] Using the DWBA, however, has shortcomings, principally in that there are relatively few investigations which give optical parameters for the deuteron at the energies relevant here and, less important is the fact that the n–p interaction is treated in a zero-range approximation; the latter can be well corrected by a normalization factor.[1] Thus, by comparing theory with experiment, some measure of the absolute spectroscopic factors is obtained.

In addition, (*p*,*d*) reactions are more selective than (*d*,*p*) reactions, since in the latter the neutron may be left in an unoccupied configuration and therefore many more levels are expected to be excited strongly. A good comparison is obtained from the Ti^{47} levels as observed in $Ti^{48}(p,d)Ti^{47}$ in this report and in $Ti^{46}(d,p)$-Ti^{47} by Rapaport.[2] Levels which in the (*p*,*d*) reaction can be ascribed to the pickup of a neutron from a lower filled shell such as the $2s_{1/2}$ or $1d_{3/2}$ shell (hole state) should not be easily seen in (*d*,*p*) reactions nor should they show a clear stripping pattern. The fact that they are found in (*d*,*p*) studies is puzzling and will be discussed later. The number of (*p*,*d*) investigations has been relatively small compared to the corresponding (*d*,*p*) investigations. This is due principally to the high negative Q value (-6 to -12 MeV) for (*p*,*d*) reactions which require correspondingly higher incident proton energies. In the present investigation using 17.5-MeV protons, deuteron spectra and angular distributions of the various deuteron groups were measured from $Q=-5.9$ to $Q=-13$ MeV. This investigation has confirmed a number of previously known results and in addition has given a large amount of spectroscopic information. Previous investigations leading to the same final states include the already mentioned work of Rapaport[2] and the (*p*,*p*′) and (*d*,*p*) work of Hansen[3] for a number of Ti isotopes. Other recent (*d*,*p*) experiments are those of Rietjens, Bilaniuk, and Macfarlane[4] and of Yntema.[5] For the pickup reaction we have the (*d*,*t*) work of Yntema[6] which demonstrates the presence of $2p$ admixture in the even-A Ti isotopes.[6] Other sources of information include the level schemes through γ-ray studies such as that of Hillman[7]

* This work supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
[1] R. H. Bassel, R. N. Drisko, and G. K. Satchler, ORNL Report No. 3240 (unpublished).

[2] J. Rapaport, thesis, Massachusetts Institute of Technology, 1963 (unpublished).
[3] O. Hansen, Nucl. Phys. **28**, 140 (1961).
[4] L. H. Th. Rietjens, O. M. Bilaniuk, and M. H. Macfarlane, Phys. Rev. **120**, 527 (1960).
[5] J. L. Yntema, Phys. Rev. **131**, 811 (1963).
[6] J. L. Yntema, Phys. Rev. **127**, 1659 (1962).
[7] M. Hillman, Phys. Rev. **129**, 2227 (1963).

on the Sc^{48} decay, in addition to much information which has been compiled in nuclear data tables.[8]

A comparison of the present results, principally excitation energies and spectroscopic factors, with the predictions of McCullen, Bayman, and Zamick[9] has been encouraging in that many of the features observed in the present experimental investigation have been well reproduced in their calculations.

EXPERIMENTAL PROCEDURE

The reactions investigated were produced by 17.5-MeV protons from the Princeton FM cyclotron, with the beam analyzed by a magnetic spectrometer in which the magnetic field was stabilized so that the beam energy did not drift by more than 10 keV. A collimating slit system selected a 30-keV portion of the beam and directed it onto the target in the small scattering chamber.[10] A telescopic arrangement of three solid state detectors defining a solid angle of approximately 5×10^{-4} sr was used to observe the reaction products from the target. The first detector, a thin surface barrier transmission detector ($\sim$50 μ), measures the specific ionization of the particles, separating protons, deuterons, tritons, etc., of a given energy through their different energy losses in this counter. The second counter, which was also of the transmission type ($\sim$500 μ), stopped all deuterons of interest and together with the first detector, served to measure their energies. The third counter was used to produce anticoincidence pulses to prevent the analyzer from accepting those pulses due to particles which reach this detector, i.e., mainly elastically scattered protons, thus considerably clearing up the spectrum of proton pile-up pulses. A more complete discussion of the detecting system and electronics used will be found elsewhere.[11,12]

The titanium isotopes were in the form of self-supporting foils of the enriched isotopes, each about 1 mg/cm^2 thick. Table I gives the thickness and isotopic composition of the targets. Since the target foils were not 100% isotopic, deuteron groups from the different isotopes often overlapped deuteron groups from the main target isotope; however, since the isotopic abundance is known for each by spectroscopic analysis, the amount of yield expected from impurities could be subtracted. The target thickness was the main limiting factor in the over-all resolution which was about 60-keV full width at half-maximum for the odd to even mass reactions and about 80 keV (fwhm) for the even to odd mass reactions.

As may be seen by reference to Table I, Ti^{48} is the principal contaminant in the other isotopes. Since the Q value to the 0.160-MeV level in Ti^{47} is known with high precision and the differential (p,d) cross section to this level is large (1.8 mb/sr at 35°), the position of this level in the various spectra and those of the ground levels of Ti^{45}, Ti^{48}, Ti^{49}, and of the first excited level in Ti^{46} for which the Q values are also known with high precision,[13] provide an excellent energy calibration for the other levels observed. These Q values are listed in Table I. In addition a Teflon target (CF_2) was used and the deuteron spectrum from the $F^{19}(p,d)F^{18}$ reaction measured to check the calibration energy. These two independent calibrations proved to be consistent and we estimate our over-all uncertainty in the excitation energies to be ±20 keV in the odd mass isotopes and ±15 keV in the even mass isotopes. The cross sections are in absolute units and the error bars generally denote the statistical uncertainties except in those cases where the separation of two or more closely spaced levels was effected, in which case the error bars have been correspondingly increased.

EXPERIMENTAL RESULTS

The energy spectra of the deuterons at a laboratory angle of 35° are shown in Fig. 1 for the $Ti^{50,48,46}(p,d)$-$Ti^{49,47,45}$ reactions and in Fig. 2 for the $Ti^{49,47}(p,d)Ti^{48,46}$ reactions. In these spectra a number of clearly resolved deuteron groups are seen. For the even-A targets, one deuteron peak stands out and dominates the spectra of Fig. 1 while for odd-A targets, a number of strong deuteron groups are observed. The two spectra shown in Fig. 2 were taken with the same gain settings and one can see at around channel 85 the deuteron group corresponding to the 0.16-MeV level of Ti^{47}. In both spectra it is due to the Ti^{48} contamination in the targets. Most of the peaks labeled appear to correspond to single levels of the particular daughter nucleus although with the present resolution it would be difficult to infer multiplicity in a peak corresponding to two levels with a separation of about 25 keV or less. An example of an unresolved group is the peak at 3.25-MeV excitation in Ti^{46} shown in Fig. 2 where from the width of the peak it can be seen that more than one level participates. A case where the multiplicity could not have been detected with the present resolution on the basis of spectra alone is that of the peak corresponding to 1.81-MeV excitation in Ti^{47}. High resolution experiments[2,3] have shown that there are two levels separated by 28 keV at this excitation energy. In the $Ti^{46}(d,p)Ti^{47}$ reaction,[2] the lower lying level of the doublet is assigned $l_n=1$ and the other possibly $l_n=3$. In the current investigation the angular distribution of the 1.81-MeV group can be explained only in terms of an $l_n=1$, plus an $l_n=2$ transition.

The angular distributions of the deuteron groups observed from the bombardment of Ti^{50} through Ti^{46}

[8] F. Ajzenberg-Selove, N. B. Gove, T. Lauritsen, C. L. McGinnis, R. Nakasima, J. Scheer, and K. Way, *Energy Levels of Nuclei: A=5 to A=257* (Springer Verlag, Berlin, 1961).
[9] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964).
[10] A. Lieber, Nucl. Instr. Methods **26**, 51 (1964).
[11] E. Kashy, Phys. Rev. **134**, B378 (1964).
[12] K. S. Thorne and E. Kashy (to be published).
[13] Tables of Nuclear Reaction Q Values, UCRL Report No. 5419 (unpublished).

TABLE I. Isotopic abundance, target thickness, and (p,d) Q values of the titanium targets.

Target / Atomic percent of	Ti^{46}	Ti^{47}	Ti^{48}	Ti^{49}	Ti^{50}
46	86.4±0.3	1.87	0.18±0.05	1.98±0.05	3.1 ±0.05
47	2.4±0.1	80.1±0.1	0.33±0.05	1.84±0.05	2.39±0.05
48	9.6±0.2	15.8	99.3 ±0.1	18.8 ±0.1	22.8 ±0.1
49	0.8±0.1	1.11	0.37±0.05	75.7 ±0.1	2.02±0.05
50	0.9±0.1	1.1	0.13±0.05	1.67±0.05	69.7 ±0.1
Target thickness (mg/cm^2)	1.09	0.837	1.042	0.975	0.887
(p,d) Q value (Ref. 13) (MeV)	−10.962 ±0.006	−6.656 ±0.006	−9.393 ±0.005	−5.917 ±0.004	−8.709 ±0.004

are shown in Figs. 3 through 7. In these figures, all the experimental data are shown, with the laboratory differential cross sections plotted versus laboratory angle. Each angular distribution is identified by the excitation energy of the corresponding level in the daughter nucleus. The angular momentum of the picked up neutron is also shown, determined principally by the systematics of the angular distribution measured, since definite similarities in the shapes of the angular distributions were observed. The excitation energies of levels obtained here are in close agreement with those in other investigations[2,3] so that there is little doubt as to the correspondence of levels observed here to those observed in earlier investigations. The excitation of a previously reported[4] level at 0.55 MeV in Ti^{47} by the (p,d) reaction appears very small, if it is excited at all. The data shown in Figs. 1 and 5 for a level at 0.58 MeV give an idea of the smallness of the cross section

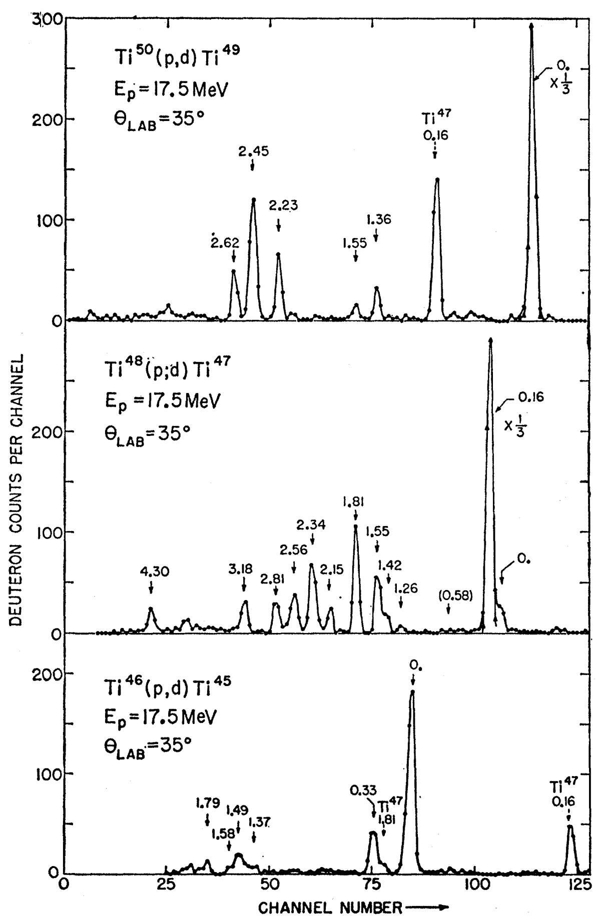

FIG. 1. Energy spectra of the deuterons from the reactions $Ti^{50}(p,d)Ti^{49}$, $Ti^{48}(p,d)Ti^{47}$, $Ti^{46}(p,d)Ti^{45}$ at a laboratory angle of 35°, using 17.5-MeV protons.

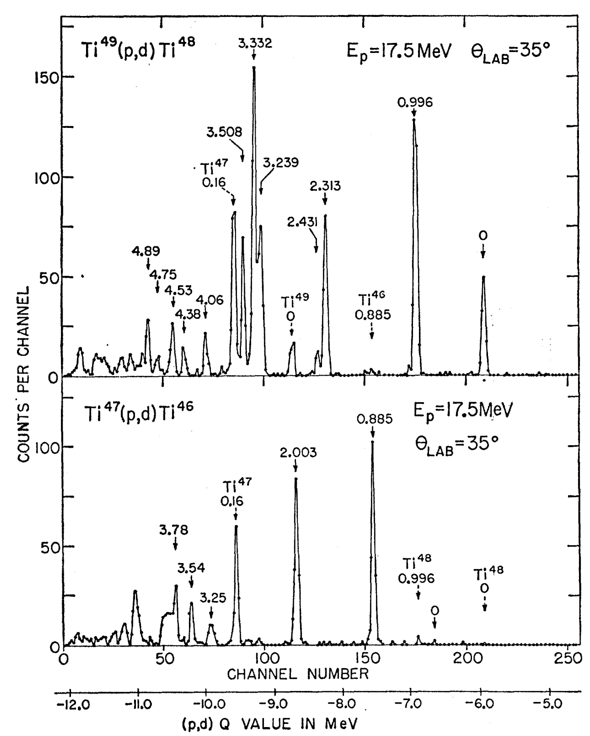

FIG. 2. Energy spectra of the deuterons from the reactions $Ti^{49}(p,d)Ti^{48}$ and $Ti^{47}(p,d)Ti^{46}$ at a laboratory angle of 35°, using 17.5-MeV protons.

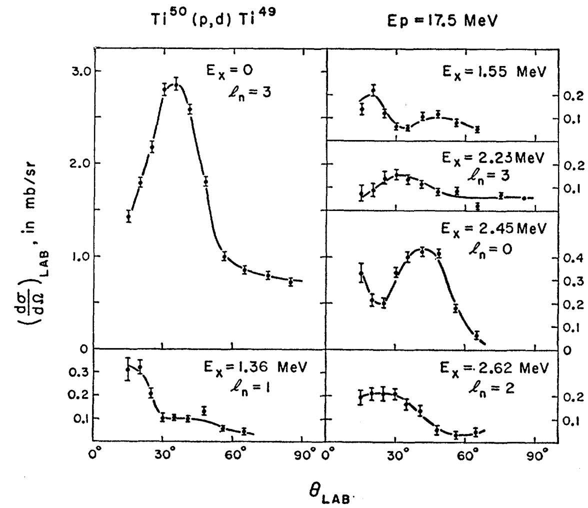

FIG. 3. Angular distributions of the deuteron groups resulting from the $Ti^{50}(p,d)Ti^{49}$ reaction.

should such a level indeed exist. Figures 3 to 7 thus represent a graphical summary of the (p,d) results, nucleus by nucleus.

The anomalous spin of Ti^{47} has consequences which can be clearly seen in the $Ti^{48}(p,d)Ti^{47}$ and $Ti^{47}(p,d)Ti^{46}$ reactions where in both cases the ground level to ground level transition is highly inhibited. This is due to the fact that the $\frac{5}{2}^-$ level of Ti^{47} appears to be mostly due to neutrons in the $(1f_{7/2})_{5/2}{}^5$ configuration which cannot be reached by removing a neutron from Ti^{48} assuming the neutron configuration in the latter is $(1f_{7/2})_0{}^6$. In both cases the reaction must go either by compound nucleus formation or by direct reaction through the pickup of a $1f_{5/2}$ neutron. The fact that no pickup pattern is observed shows that Ti^{47} has a negligible amount of $1f_{5/2}$ in the ground-state wave function and is discussed in the next section.

A list of the excitation energies of levels observed together with the maximum value of their (p,d) differential cross section and the angle at which this maximum is observed is given in Table II for all the transitions observed.

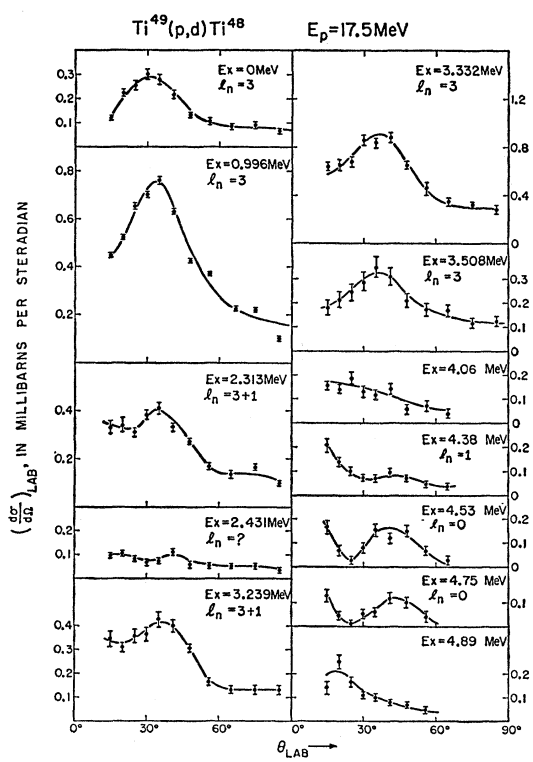

FIG. 4. Angular distributions of the deuteron groups resulting from the $Ti^{49}(p,d)Ti^{48}$ reaction.

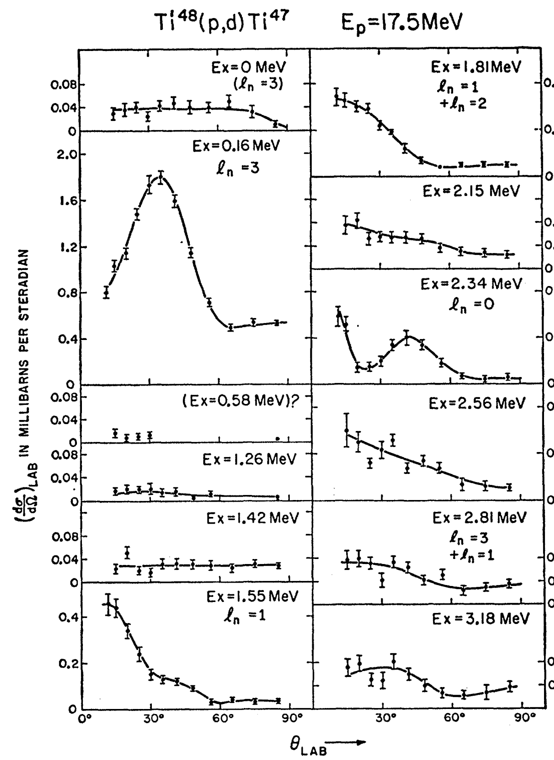

FIG. 5. Angular distributions of the deuteron groups resulting from the $Ti^{48}(p,d)Ti^{47}$ reaction.

ANALYSIS AND DISCUSSION

$l_n=1$ Transitions

In the transitions from the $\frac{7}{2}^-$ ground level of Ti^{49} to Ti^{48} levels with $J^\pi=2^+$, 3^+, 4^+, and 5^+, angular momentum and parity conservation allow more than one orbital angular momentum value for the picked up neutron; i.e., in this case, both $l_n=3$ and $l_n=1$ are possible. Similarly, in $Ti^{47}(p,d)Ti^{46}$ reaction, levels in

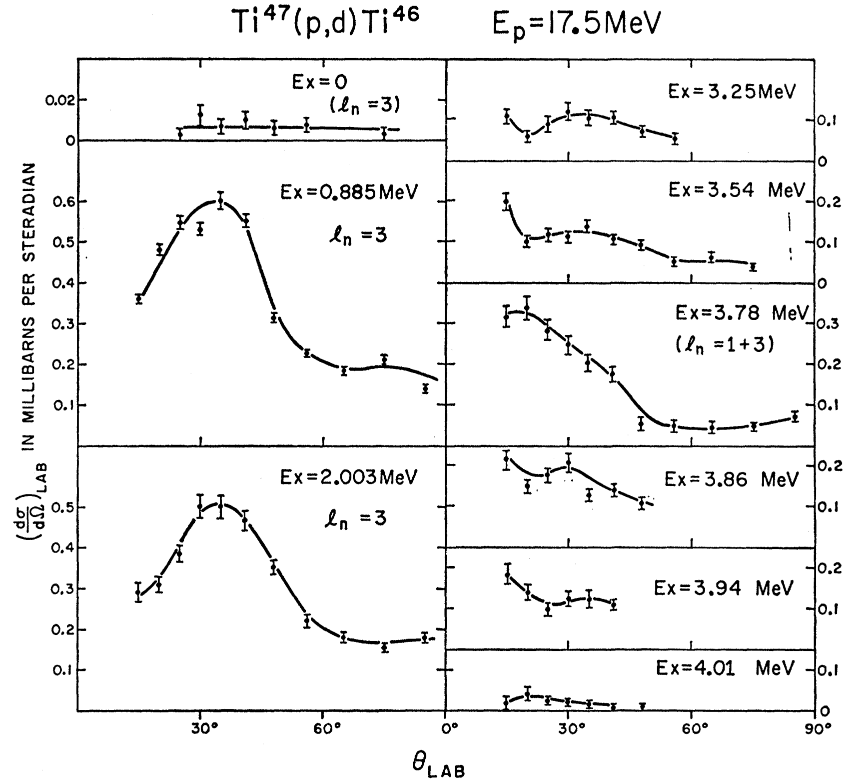

FIG. 6. Angular distributions of the deuteron groups resulting from the $Ti^{47}(p,d)Ti^{46}$ reaction.

Ti^{46} with $J^\pi = 1^+, 2^+, 3^+, 4^+$ can have angular distributions with both $l_n = 1$ and $l_n = 3$. The extent to which these transitions proceed by $l_n = 1$ therefore represent a measure of the p-wave admixture in initial and final states. In this investigation $2p_{3/2}$ admixture appears to be the dominant ground level admixture to the $1f_{7/2}$ configuration. This is understandable since the $2p_{3/2}$ lies much lower in excitation energy than the $2p_{1/2}$ single-particle state. The latter lies consistently about 2 MeV above the $2p_{3/2}$ single-particle state.[14]

The evidence of $2p_{3/2}$ admixture in the ground-level wave functions of Ti^{48} and Ti^{50} is clear from the results of the $Ti^{50,48}(p,d)Ti^{49,47}$ reactions as a number of levels due to $l_n = 1$ pickup are observed. In the $Ti^{50}(p,d)Ti^{49}$ reaction such a transition to a level at 1.36-MeV excitation in Ti^{49} is observed in agreement with $Ti^{48}(d,p)Ti^{49}$ results for this level (given in Ref. 4 as 1.38 MeV) where it is found to be the strongest $l_n = 1$ transition and appears to carry most of the $2p_{3/2}$ single-particle spectroscopic factor.[4] Also in the same investigation,[4] a strong $l_n = 1$ transition is observed at an excitation energy of 1.72 MeV, with a value of $(2J+1)\theta^2$ equal to 0.45 that of the 1.38-MeV transition. In the present (p,d) investigation the 1.72-MeV level is not reached or at least one can assign an upper limit for its (p,d) cross section strength as 0.05 of that to the 1.36-MeV Ti^{49} level. This large discrepancy in excitations by (d,p) and (p,d) must hold a clue as to the difference between these levels. One possibility is that the 1.72-MeV level has spin $\frac{1}{2}^-$ so that with the small $2p_{1/2}$ admixture expected in the Ti^{50} ground level, this level would not be seen in the (p,d) reaction. This interpretation is consistent with measurements of circular polarization of γ ray following capture of polarized neutrons in Ti^{48} carried out by G. Trumpy[15] and later by J. Vervier[16] which indeed show that the 1.36-

[14] R. H. Nussbaum, Rev. Mod. Phys. **28**, 423 (1956).

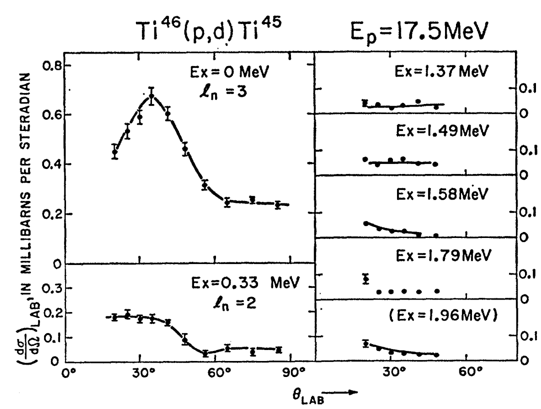

FIG. 7. Angular distributions of the deuteron groups resulting from the $Ti^{46}(p,d)Ti^{45}$ reaction.

[15] G. Trumpy, Nucl. Phys. **2**, 664 (1957).
[16] J. Vervier, Nucl. Phys. **26**, 10 (1961).

TABLE II. Summary of results for (p,d) reactions in the Ti isotopes.

E_x[a] (MeV)	Q (MeV)	l_n	J^π[b]	J^π[c]	$(\sigma_{lab})_{max}$ (mb/sr)	$(\theta_{lab})_{max}$	S
			$Ti^{46}(p,d)Ti^{45}$				
0.	−10.96	3	$\frac{7}{2}^-$	$\frac{7}{2}^-$	0.67	35°	2.5
0.33	−11.29	2	$\frac{3}{2}^+$	...	0.20	25°	0.7
1.37	−12.33	...	...	...	0.03	d	...
1.49	−12.45	...	...	...	0.06	d	...
1.58	−12.54	...	...	...	0.05	...	...
1.79	−12.75	...	...	...	0.08	d	...
			$Ti^{47}(p,d)Ti^{46}$				
0	−6.660	(3)	√	0^+	0.007	d	≤0.007
0.885	−7.545	3	√	2^+	0.60	35°	0.6
2.003	−8.663	3	√	4^+	0.50	35°	0.8
3.25[e]	−9.91	...	...	...	0.12	30°	...
3.54	−10.20	...	...	...	0.20	15°	...
3.78	−10.44	1 plus	...	...	0.26	15°	0.4
		3		...	0.11	35°	0.3
3.86	−10.52	...	...	...	0.20	30°	...
3.94	−10.60	...	...	...	0.13	d	...
4.01	−10.67	...	...	...	0.02	d	...
			$Ti^{48}(p,d)Ti^{47}$				
0	−9.393	(3)	...	$\frac{5}{2}^-$	0.04	d	≤0.08
0.16	−9.553	3	$\frac{7}{2}^-$	$\frac{7}{2}^-$	1.82	35°	3.8
(0.58)	(−9.97)	...	...	...	≤(0.01)	(d)	...
1.26	−10.65	...	...	...	0.015	d	...
1.42	−10.81	...	...	...	0.03	d	...
1.55	−10.94	1	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0.45	15°	1.1
1.81[e,f]	−11.20	1	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0.12	15°	0.3
		2	$\frac{3}{2}^+$	...	0.20	25°	0.7
2.15	−11.54	...	...	...	0.08	20°	...
2.34	−11.73	0	$\frac{1}{2}^+$	$\frac{1}{2}^+$	0.20[g]	41°	...
2.56	−11.95	...	...	...	0.12	15°	...
2.81[e]	−12.20	1	$(\frac{3}{2}^-)$	$(\frac{3}{2},\frac{1}{2})^-$	~0.06	~15°	0.5
		3	$(\frac{7}{2}^-)$	$(\frac{7}{2}^-)$	~0.05	~35°	0.4
3.18	−12.57	...	...	...	0.08	15°	...
			$Ti^{49}(p,d)Ti^{48}$				
0	−5.917	3	√	0^+	0.30	35°	0.2
0.996	−6.913	3	√	2^+	0.75	35°	0.7
2.313	−8.230	3 plus	√	4^+	0.40	35°	0.5
		1			0.10	15°	0.04
2.431	−8.348	...	...	(1,2)	0.1	d	...
3.239	−9.156	3 plus	√	4^+	0.41	35°	0.7
		1			0.10	15°	0.06
3.332	−9.249	3	√	6^+	0.90	35°	1.8
3.508	−9.425	3	(6^+)	5^+–8^+	0.35	35°	0.7
4.06	−9.98	...	...	...	...	...	...
4.38	−10.30	1	...	...	0.21	15°	0.3
4.53	−10.45	0	$(4,3)^+$	...	0.14[g]	41°	...
4.75	−10.67	0	$(3,4)^+$	...	0.12[g]	41°	...
4.89	−10.81	...	...	...	0.25	20°	...
			$Ti^{50}(p,d)Ti^{49}$				
0	−8.71	3	$\frac{7}{2}^-$	$\frac{7}{2}^-$	2.70	35°	4.4
1.36	−10.07	1	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0.31	15°	0.4
1.55	−10.26	...	...	...	0.2	20°	...
(1.72)	...	...	$(\frac{1}{2}^-)$	$\frac{1}{2}^-$	≤0.01	...	...
2.23	−10.94	3	$\frac{7}{2}^-$	...	0.15	30°	0.6
2.45	−11.16	0	$\frac{1}{2}^+$	...	0.42[g]	41°	...
2.62	−11.33	2	$\frac{3}{2}^+$	...	0.20	25°	0.7

[a] $E_x \pm 20$ keV for Ti^{49}, Ti^{47}, Ti^{45}, ±15 keV for Ti^{48}, Ti^{46}.
[b] Present assignments; the check mark (√) indicates that the present results are consistent with previous assignments.
[c] Previous assignments.
[d] Approximately isotropic.
[e] Doublet.
[f] Using the energies from Ref. 2, the 1.788-MeV level has $l_n=1$ and the 1.816-MeV level has $l_n=2$. See also Ref. 4.
[g] σ given at secondary maximum of $l_n=0$ distribution.

and 1.72-MeV levels have spins $\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively, and contradicts the interpretation of Rietjens *et al.*[4] who interpret the 1.72 MeV as a $\frac{3}{2}^-$ level. It also shows that neglect of $2p_{1/2}$ admixture in Ti^{50} ground state is justified.

In the $Ti^{48}(p,d)Ti^{47}$ reaction two $l_n=1$ transitions are

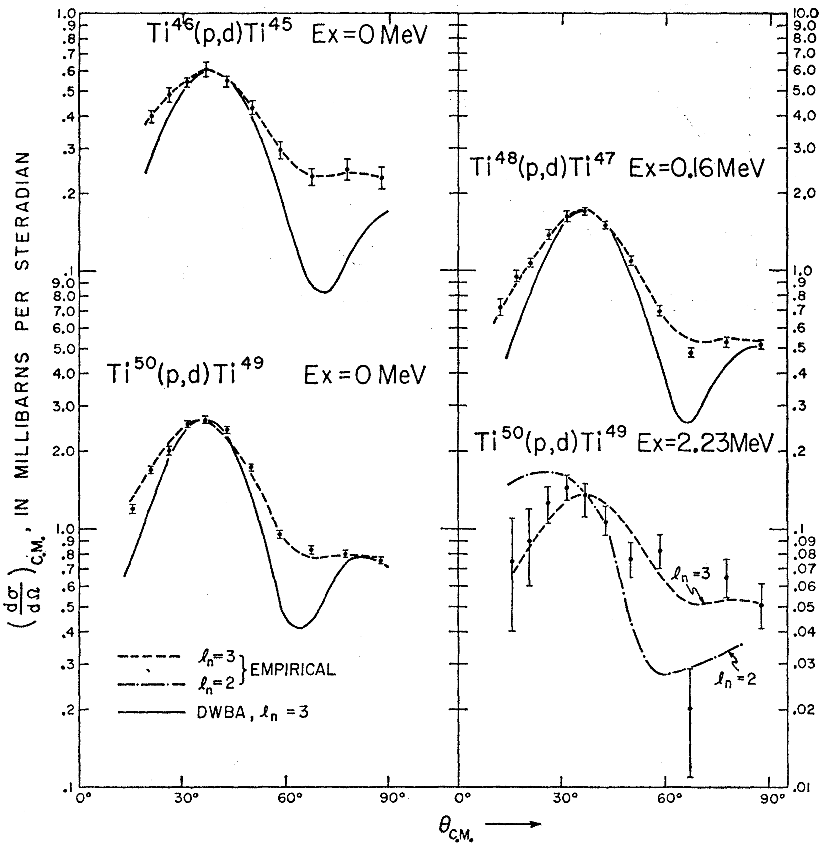

FIG. 8. Angular distributions for transitions corresponding to $l_n=3$ pickup in even mass targets. The dashed curves represent the empirical curves shown in Fig. 9.

observed, a relatively strong $l_n=1$ corresponding to 1.55-MeV excitation in Ti^{47} and a weaker group, which is due in part to $l_n=1$, at 1.81-MeV excitation. For the 1.55-MeV level there is agreement with earlier Ti^{46} $(d,p)Ti^{47}$ results.[4,2] The 1.81-MeV group is resolved by Rapaport[2] as a close lying doublet at 1.788- and 1.816-MeV excitation. The 1.788 MeV is excited in the (d,p) reaction by a strong $l_n=1$ transition and the 1.816 MeV given as probably due to $l_n=3$. Earlier work by Hansen[3] has also shown that a close lying doublet existed at energies of 1.793 and 1.823 MeV in Ti^{47}. The 1.81-MeV (p,d) yield was thus decomposed by subtracting different $l_n=1$ contributions from the group using the $l_n=1$ angular distribution to the 1.55-MeV level as standard. It was found that when $25\pm5\%$ of the 1.55-MeV cross section was subtracted, the angular distribution remaining had a shape typical of $l_n=2$ pickup as opposed to $l_n=3$, in agreement with recent (d,t) results.[6] The ratio of peak differential cross section of the 1.81 ($l_n=1$) to the 1.55 is 0.25 ± 0.05 in the present (p,d) results and 0.31 in the (d,p) reaction,[2] indeed quite close. Since $J^\pi=\frac{1}{2}^-$ levels would not be expected to be strongly excited by the (p,d) reaction, this is good evidence for assigning both the 1.55- and 1.81-MeV levels $J^\pi=\frac{3}{2}^-$. No clear $l_n=1$ transitions were observed in the $Ti^{46}(p,d)Ti^{45}$ reaction. In the $Ti^{49}(p,d)Ti^{48}$ the 4.38-MeV level of Ti^{48} appears to be the result of an $l_n=1$ pickup.

$l_n=3$ Transitions, Even A to Odd A

Having established that some of the titanium isotopes have ground levels with $2p_{3/2}$ admixture, it was necessary to find a way of determining to what extent such transitions as in the odd-A to even-A Ti isotopes $J^\pi=\frac{7}{2}^-$ to $J^\pi=2^+$ or $J^\pi=\frac{5}{2}^-$ to $J^\pi=4^+$ take place by $l_n=3$ and by $l_n=1$ pickup, since both l values are allowed. The DWBA calculations of the angular distributions could not be used for this purpose to give the shape of an $l_n=3$ (p,d) transition, since these calculations did not give a good fit to those transitions where angular momentum conservation showed that only $l_n=3$ was allowed, as can be seen in Fig. 8. Instead, using the angular distribution data of the $Ti^{50,49,46}$-$(p,d)Ti^{49,48,45}$ ground level to ground level transitions and the $Ti^{48}(p,d)Ti^{47}$ transition to the 0.16-MeV Ti^{47} level, three empirical curves for pure $l_n=3$ transitions over a range of Q values were drawn and are shown in Fig. 9. These curves are normalized to 1.0 mb/sr peak cross section. Also shown in Fig. 9 is an $l_n=1$ curve obtained from the $Fe^{56}(p,d)Fe^{55}$ reaction which is also normalized to a value of 1 mb/sr peak cross section. The values of the parameters α and β shown

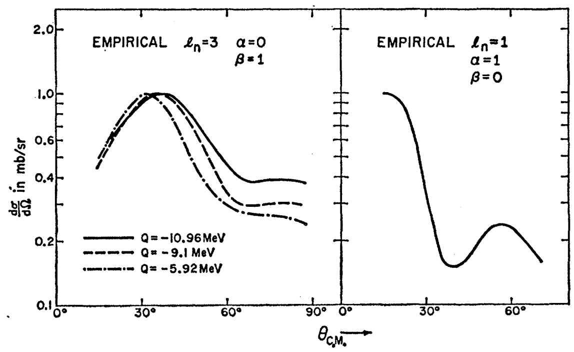

FIG. 9. Empirical angular distributions for $l_n=3$ deuteron pickup ($\alpha=0$, $\beta=1$) for three different Q values in the titanium isotopes, and for $l_n=1$ pickup ($\alpha=1$, $\beta=0$).

in Fig. 9 are used to indicate the relative $l_n=1$ and $l_n=3$ components in the angular distribution, i.e., $\alpha=0$ $\beta=1$ represents an $l_n=3$ angular distribution normalized to have a peak cross section of 1 mb/sr and no $l_n=1$ contribution.

The angular distributions for $l_n=3$ transitions from the even to odd isotopes are shown in Fig. 8, together with the empirical $l_n=3$ curves of Fig. 9, shown as the dashed curves. DWBA calculations were carried out using the Oak Ridge DW code.[1] The optical parameters used were taken from an investigation by Perey[17] for the protons and by C. M. Perey and F. G. Perey[18] for the deuterons. Deuteron optical parameters for deuteron energies relevant in the present investigation are not well known and have been assumed here to be constant over the whole range of deuteron energy involved. The theoretical maximum of the pickup differential cross section is plotted in Fig. 10 versus (p,d) Q value,[19] and was used to extract the spectroscopic factors. Also shown in Fig. 10 are similar curves for $l_n=1$ and $l_n=2$ transitions calculated with the same optical parameters. The resulting spectroscopic factors are tabulated in Table II, and for $l_n=3$ transitions, shown graphically in Fig. 11 where they are compared to theoretical prediction by McCullen, Bayman, and Zamick.[9] In these calculations they inferred two body matrix elements from the Sc^{42} spectrum, and assumed that the Ti isotopes could be described by a pure $(1f_{7/2})^n$ configuration.

A transition of special interest is that of the 2.23-MeV level in Ti^{49}. Its angular distribution is shown in Fig. 8. The transition is best explained in terms of an $l_n=3$ pickup and is probably to a $\frac{7}{2}^-$ level of Ti^{49}. In a previous $Ti^{50}(d,t)Ti^{49}$ investigation[6] with low resolution, the 2.23-, 2.45-, 2.62-MeV levels were unresolved, and the whole group was assumed to have an $l_n=3$ distribution, giving this transition an unrealistically large $l_n=3$ spectroscopic factor. The fact that the whole group was assigned as $l_n=3$ is probably due to the l value of the levels involved, $l_n=0$ for the 2.45-MeV level and $l_n=2$ for the 2.62-MeV level, so that the sum of all three levels can easily be mistaken for an $l_n=3$ transition even though the $l_n=3$ is the weakest of the three transitions. In the $Ti^{48}(p,d)Ti^{47}$ reaction, the 2.56- and 2.81-MeV groups are due to a number of levels.[2] In the 2.81-MeV group where 2 transitions, an $l_n=3$ and an $l_n=1$ are involved,[2] an estimate of the relative cross sections has been made. These are listed in Table I, together with the resulting spectroscopic factors.

The present (p,d) results on even Ti isotopes contradicts earlier (d,t) results indicating that strong $l_n=3$ transitions occur at about 2-MeV excitation.[6] The $Ti^{50}(p,d)Ti^{49}$ case has just been discussed. In the Ti^{48}-$(p,d)Ti^{47}$, again a number of levels appear to be involved; e.g., the 2.4-MeV $l_n=3$ inferred from the (d,t) results[6] appears to be due principally to an $l_n=0$, $J^\pi=\frac{1}{2}^+$ level at 2.34 MeV. In the $Ti^{46}(p,d)Ti^{45}$ reaction, it is clear that a number of levels are excited close to 2-MeV excitation, so that any statement that an $l_n=3$ transition is involved appears unjustified.

The agreement between theory and experiment for the $l_n=3$ transitions from even-A Ti isotopes is quite encouraging and is shown in Fig. 11. The 2.23-MeV level of Ti^{49} is of particular interest since it is one of the four possible $\frac{7}{2}^-$ levels of Ti^{49} which can be ascribed to the $(1f_{7/2})^n$ configuration. Both its excitation energy and spectroscopic factors are in agreement with the theoretical prediction as shown in Fig. 11. In the Ti^{48}-$(p,d)Ti^{47}$ reaction, the transition to the ground level, 0.58-MeV level (if it exists), 1.26-MeV level, and 1.42-MeV level are all approximately isotropic and have very small cross sections. The ground level spin of Ti^{47} is known[20] to be $\frac{5}{2}^-$ and is well described by $(1f_{7/2})^n$

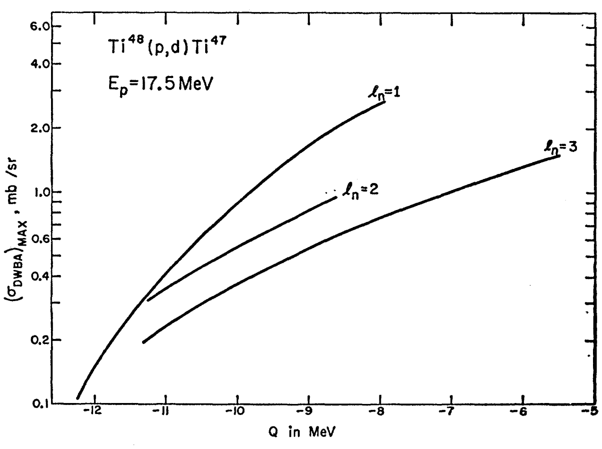

FIG. 10. Dependence of the peak cross section on Q value as predicted by DWBA calculations for $l_n=3$, $l_n=2$, and $l_n=1$.

[17] F. G. Perey, Phys. Rev. **131**, 745 (1963).
[18] C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).
[19] The optical parameters were: for the neutrons, $r_0=1.25$ F $a=0.65$ F. For the protons, $V=48$ MeV, $W=0$, $r_0=1.25$ F, $r_c=1.25$ F, $a=0.65$ F, $r_0'=1.25$ F, $a'=0.47$ F, $W'=44$ MeV. For the deuterons, $V=77.6$ MeV, $W=0$, $r_0=1.3$ F, $r_c=1.3$ F, $a=0.73$ F, $r_0'=1.35$ F, $a'=65$ F, $W'=90$ MeV.
[20] C. D. Jeffries, Phys. Rev. **92**, 1262 (1953).

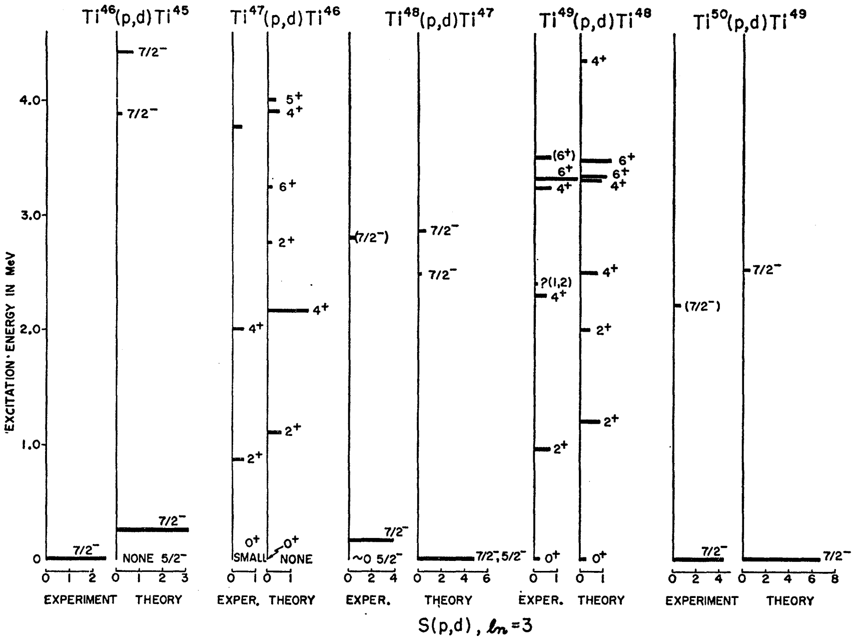

FIG. 11. Comparison of experimental and theoretical spectroscopic factors for $l_n=3$ for the reactions: $Ti^{46}(p,d)Ti^{45}$, $Ti^{47}(p,d)Ti^{46}$, $Ti^{48}(p,d)Ti^{47}$, $Ti^{49}(p,d)Ti^{48}$, $Ti^{50}(p,d)Ti^{49}$.

configuration, and it appears quite possible that the others belong to the same configuration with spins $\frac{1}{2}^-$, $\frac{5}{2}^-$, $\frac{9}{2}^-$, as predicted by McCullen, Bayman, and Zamick.[9] In none of these cases would one expect a direct reaction pattern unless the Ti^{48} ground level were admixed with the corresponding single-particle configuration. Finally, the present results appear to cast some doubt as to the assignment of $1f_{5/2}$ single-particle level at 2.5 MeV in Ti^{49} as given by Schiffer, Lee, and Zeidman[21] in a low resolution (d,p) investigation.

$l_n=3$ Transitions: Odd A to Even A

The analysis of the deuteron angular distributions from the $Ti^{49,47}(p,d)Ti^{48,46}$ reactions for the cases where the transitions were due principally to $l_n=3$ pickup was done by comparing the experimental distributions with the empirical curves of Fig. 9. The results for eight strong transitions are shown in Fig. 12, with the parameters α and β representing the relative $l_n=1$ and $l_n=3$ contributions of the empirical cross sections shown in Fig. 9. Two of the angular distributions, those to the 4^+ levels in Ti^{48} at 2.313 and 3.239 MeV, could not be explained without a relatively large $l_n=1$ contribution. For the remaining angular distribution shown in Fig. 12, the $l_n=1$ contribution was either forbidden by angular momentum conservation or else resulted in a small fraction of the observed cross section.

The energy level structure of Ti^{48} is relatively well known from a number of studies,[8] including the recent investigation by Hillman of the decay of Sc^{48} which summarizes many of the previous results.[7] The present results from the (p,d) investigation are in good agreement with previous results and provide a measure of the spectroscopic factors of the transitions. Two of the results should be noted: first, the deuteron angular distribution to the 2.431-MeV level of Ti^{48} does not exhibit a pattern identifiable with a particular l_n value and is in fact close to isotropic; second, the Ti^{48} level at 3.508 shows definite $l_n=3$ pickup pattern completely devoid of any $l_n=1$ admixture. In the work of Hillman,[7] the spin of this latter level is limited to 5, 6, 7, or 8. The present results clearly eliminate the possibility of $J=8$ for this level and indicate that the level is a strong candidate for the 6^+ level predicted in the calculations by McCullen, Bayman, and Zamick.[9] A graphical comparison of spectroscopic factors and energies with these theoretical predictions is shown in Fig. 11 for the $Ti^{49}(p,d)Ti^{48}$ reaction. The agreement is quite good, especially since in the calculations the rather appreciable $2p_{3/2}$ admixture in the ground-level wave functions is neglected. There is, however, a large discrepancy in the second 2^+ level of Ti^{48}. In the prediction, it lies below the first 4^+ level and should be quite strongly excited by the (p,d) reaction. Experimentally, if one assumes that the 2^+ level is the 2.431-MeV level, it lies above the known 2.313 MeV, 4^+ level and is relatively weak.

The transition from Ti^{47} to the ground level of Ti^{46} is strongly inhibited. The angular distribution is close to isotropic, with a differential cross section of 0.007 mb/sr. Since for the Ti^{47} target, $J^\pi=\frac{5}{2}^-$, these results indicate that the $1f_{5/2}$ component in the Ti^{47} ground

[21] J. P. Schiffer, L. L. Lee, and B. Zeidman, Phys. Rev. **115**, 427 (1959).

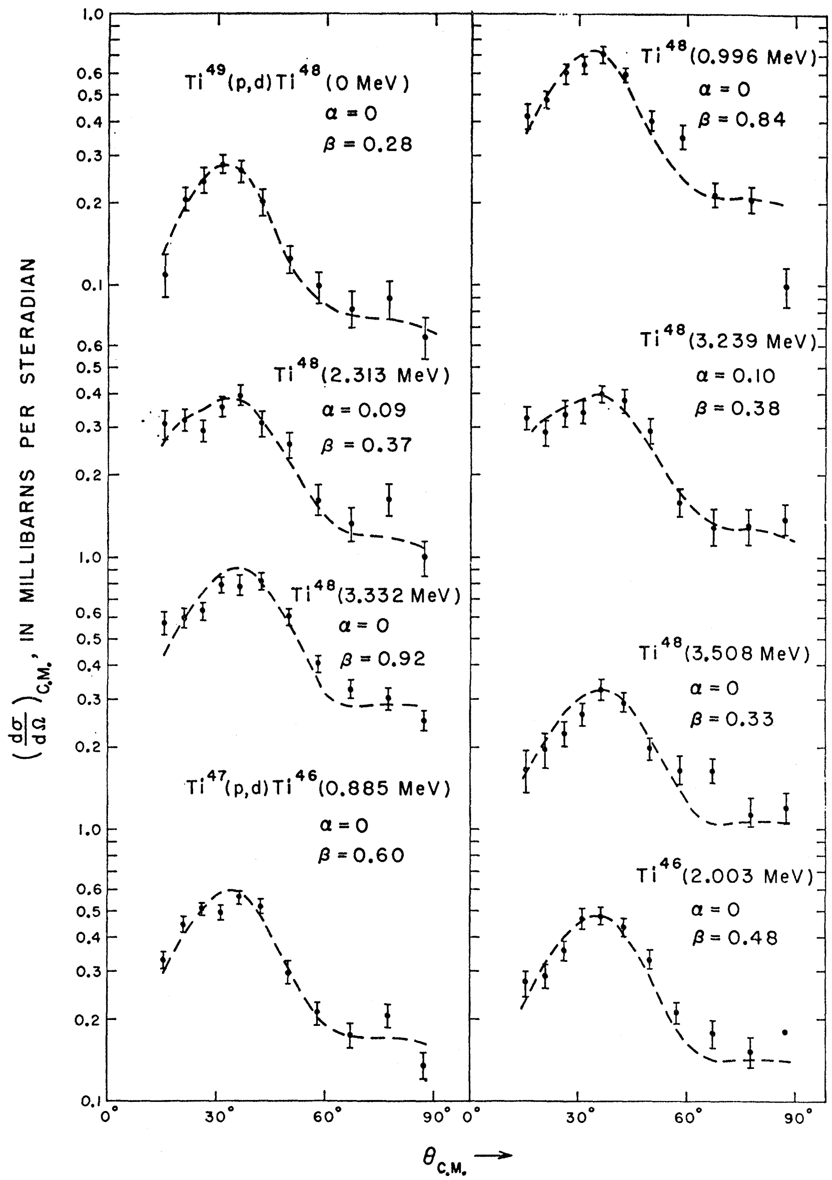

FIG. 12. Angular distributions, plotted on a semilogarithmic scale for the $l_n=3$ pickup in odd-mass targets; the dashed curves represent the empirical curves from Fig. 9.

level is indeed small, i.e., if one writes

$$\psi(\mathrm{Ti}^{47})=\alpha(1f_{7/2})_{5/2}{}^{n}+\beta[(1f_{7/2})_{0}{}^{n-1}(1f_{5/2})]_{5/2},$$

the present results give, using the DW calculation to to get S and assuming the level is excited by a direct reaction, $\beta^2/\alpha^2\leq 0.007$, indeed quite small. This is easily understood since the single-particle $1f_{5/2}$ in Ti^{47} must lie well above 4-MeV excitation energy.[2] This upper limit of 0.007 is optimistic since much of the excitation of the $\frac{5}{2}^-$ level appears to go into formation of the compound nucleus. The compound nucleus appears to contribute significantly to the cross section in the (d,p) reaction as can be inferred from (d,p) results with 7.0-MeV deuterons[2] which give $\beta^2/\alpha^2\leq 0.04$ and (d,p) results with 7.8-MeV deuterons[4] which give $\beta^2/\alpha^2\leq 0.03$, both considerably larger upper limits than in the present (p,d) case. The transition to the 2^+ and 4^+ levels of Ti^{46} at 0.885 and 2.003 MeV are well explained in terms of the pickup of an $1f_{7/2}$ neutron. Their angular distributions, which are shown in Fig. 12, did not require any appreciable $l_n=1$ admixture. One additional group of deuterons which appears to be due to a combination of $l_n=1$ and $l_n=3$ has been observed at 3.78 MeV in Ti^{46}. This group appears to be a single level but could easily be a closely spaced doublet.

The comparison of theory and experiment for the $\mathrm{Ti}^{47}(p,d)\mathrm{Ti}^{46}$ reaction is shown in Fig. 11, where a rather large discrepancy exists for the spectroscopic factor of the 4^+ level at 2.003 MeV. Figure 13 shows a plot of the total observed $l_n=3$ spectroscopic factors versus neutron number in excess of 20. From these data, it appears that a large fraction of the $l_n=3$ yield has not been detected in the $Ti^{47}(p,d)Ti^{46}$ reaction, which may be due to anomalous spin of Ti^{47}. These $l_n=3$ spectroscopic factors tend to be lower than theoretically predicted as shown in Fig. 11. Apart from the uncertainty in the optical parameters, this reflects the large p-wave admixture in the Ti isotopes as measured by the $l_n=1$ spectroscopic factors listed in Table II.

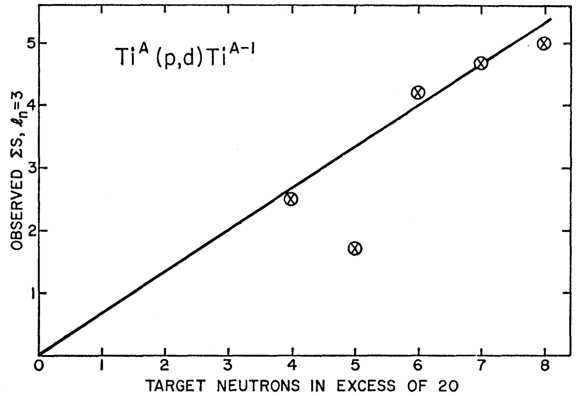

FIG. 13. Plot of the sum of the spectroscopic factors for $l_n=3$ transitions for each titanium target versus the number of neutrons in excess of 20 in that isotope.

$l_n=0$ and $l_n=2$ Transitions

A number of even l_n transitions have been observed, corresponding to the pickup of a neutron from a filled shell, i.e., the $1d_{3/2}$ and $2s_{1/2}$ shells. These transitions are labeled as $l_n=0$ or $l_n=2$ in the angular distributions shown in Figs. 3 through 7, and are also listed in Table II. An interesting feature of these $l_n=0$ and $l_n=2$ transitions is demonstrated by the Ti^{47} level at 2.34 MeV. It is excited by $l_n=0$ in the $Ti^{48}(p,d)Ti^{47}$ reaction as shown in Fig. 5. Recent results by Rapaport[2] show that in the $Ti^{46}(d,p)Ti^{47}$ reaction, an $l_n=0$ transition showing a typical direct reaction angular distribution is observed at 2.361 MeV, and within the accuracy of our calibration, it corresponds to the 2.34-MeV level. One possible explanation is that the level can be expressed as a linear combination of $(2s_{1/2})^{-1}$ and $(3s_{1/2})^1$ so that both (p,d) and (d,p) can show direct reaction patterns. If we assume that the $3s_{1/2}$ component is negligible, the stripping could still take place if part of the Ti^{46} ground level had both neutrons missing from the $2s_{1/2}$ shell so that the stripping reaction could take place by a $2s_{1/2}$ neutron. Since the $3s_{1/2}$ single-particle energy level is at 5 MeV in Ti^{47}, the second case may be the more likely one. A similar argument for $l_n=2$ could explain $Ti^{46}(d,p)Ti^{47}$ stripping to the 1.81-MeV level of Ti^{47}. In this case, the argument is much stronger, since the level reached by (p,d) has $J^\pi=\frac{3}{2}^+$ while the next $l_n=2$ shell corresponds to $J^\pi=\frac{5}{2}^+$, i.e., the $2d_{5/2}$ shell. This indicates that part of the ground-state wave function of Ti^{46} must have a term with a $(1d_{3/2})^{-2}$ component. This interaction of $2d_{3/2}$ and $1f_{7/2}$ neutrons requires that the matrix element $\langle (d_{3/2})^2 | V_{12} | (f_{7/2})^2 \rangle$ be important, thus showing evidence of p_3 or p_5 coefficients in the internucleon potential.

In each of the even-A to odd-A reactions, one $l_n=2$ transition was observed, corresponding to levels at 0.33, 1.81, and 2.62 MeV in Ti^{45}, Ti^{47}, and Ti^{49}, respectively; the corresponding (p,d) Q values are -11.29, -11.20, and -11.33 MeV. In the $Ti^{48}(p,d)Ti^{47}$ case, the deuteron yield to the 1.81-MeV group was decomposed into two closely spaced levels, as had been established in two earlier experiments,[2,3] where the separation was given as 28 keV. For the lower level, Rapaport assigned $l_n=1$ while for the considerably

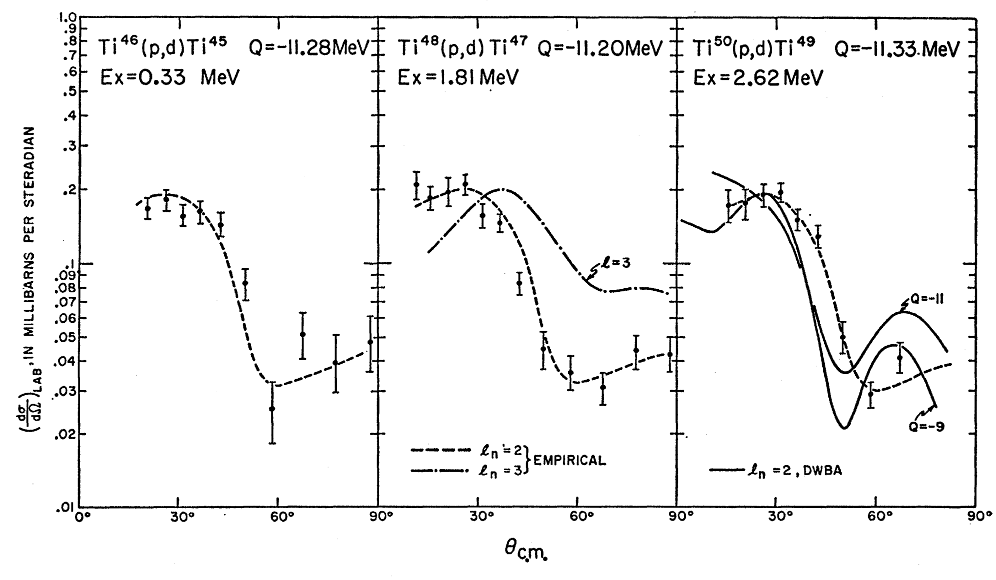

FIG. 14. Angular distribution of deuterons from $Ti^{50}(p,d)Ti^{49}$, $Ti^{48}(p,d)Ti^{47}$, $Ti^{46}(p,d)Ti^{45}$ which show $l_n=2$ stripping. The data for the 1.81-MeV group (doublet) in Ti^{47} represent the remaining cross section after subtraction of the $l_n=1$ component. The dashed curves are empirical $l_n=2$ curves from the other two $l_n=2$ distributions. Also shown for comparison is an empirical $l_n=3$ curve.

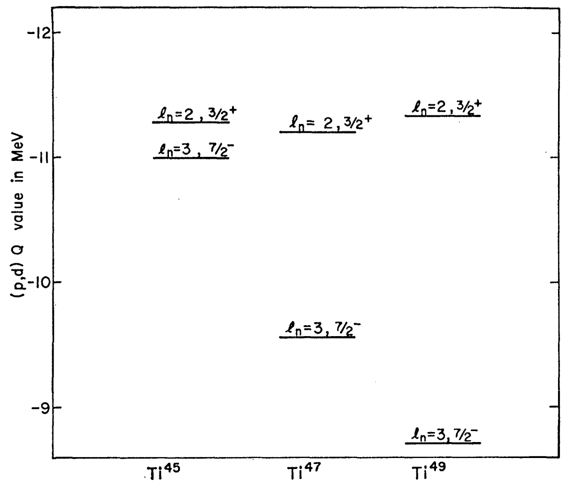

FIG. 15. Systematics of (p,d) Q values of the main $l_n=3$ and of the $l_n=2$ transitions for the even-A to odd-A Ti isotopes.

weaker upper level a probable assignment of $l_n=3$ was given. The (p,d) angular distribution, after subtraction of $l_n=1$ contribution, could not be explained by $l_n=3$ pickup but rather $l_n=2$ gave good agreement with the data. The resultant angular distribution is shown in Fig. 14 where the dashed curve is taken from the Ti^{45} 0.33-MeV angular distribution and the Ti^{49} 2.62-MeV angular distribution. Also shown in Fig. 14 are two DW calculations for $l_n=2$, where only qualitative agreement in shape is found between data and theory. We note that the assignment of the upper level in the 1.81-MeV doublet (given at 1.816 MeV in Ref. 9) as $l_n=2$ results in considerably better agreement between theoretical predictions and experiment for the $Ti^{46}(d,p)Ti^{47}$ reaction.[9] The three (p,d) Q values of $l_n=2$ transitions are within 130 keV of each other. Furthermore, the peak differential cross section is approximately the same in each case, i.e., ~0.20 mb/sr. This indicates that the energy required to remove a $1d_{3/2}$ neutron from the core is approximately unaffected by the number of $1f_{7/2}$ neutrons outside the core. This does not appear to be the case for the removal of a proton.[22] It would have been very helpful to find the $l_n=2$ transitions in the odd to even isotopes. However, the multiplicity of levels expected, 4, plus the small cross section have made this impossible with the present resolution. In the (p,d) reaction on Sc^{45}, the Q value for the pickup of $l_n=2$ has a value around -10.6 MeV quite close to the value observed here for even Ti isotopes.[11] The low-lying 0.33-MeV position of the $\frac{3}{2}^+$ level of Ti^{45} indicates that the pairing energy gained by leaving the $1f_{7/2}$ neutron coupled comes close to compensating for the loss in the $1d_{3/2}$ pairing energy in addition to the $1f_{7/2}-1d_{3/2}$ energy difference. The (p,d) Q values of the $l_n=2$ and of the main $l_n=3$ transitions from the even-A to odd-A

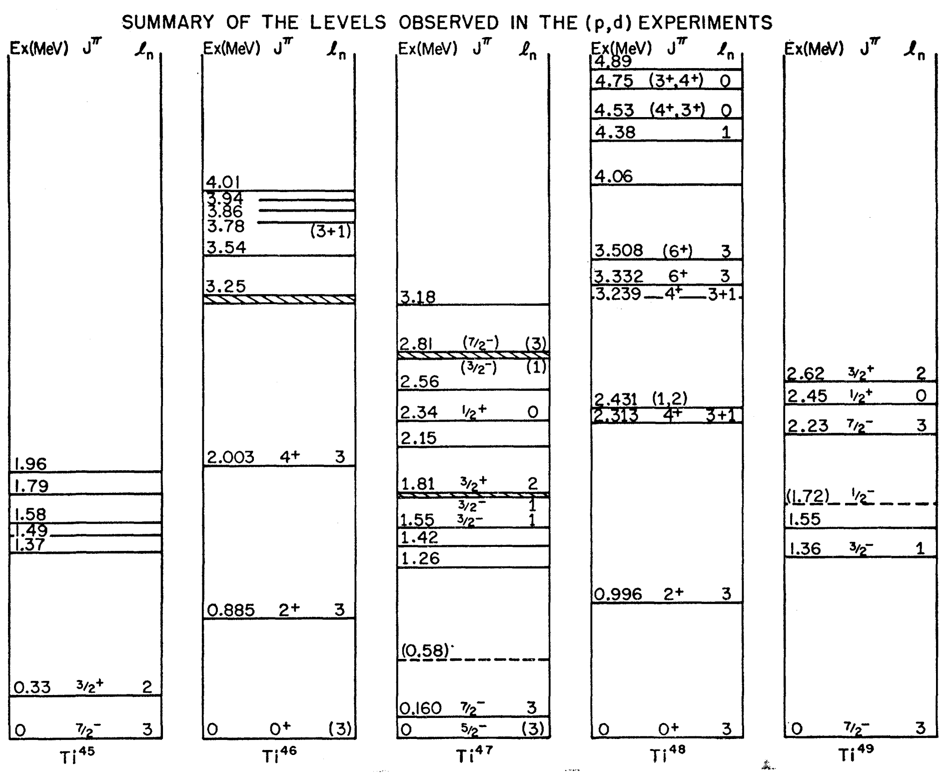

FIG. 16. Energy level diagrams showing l values, angular momenta, and parities of levels in Ti^{49}, Ti^{48}, Ti^{47}, Ti^{46}, Ti^{45}.

[22] J. L. Yntema (to be published).

isotopes are shown in Fig. 15 and indicate the possibility that the spin of Ti^{43} is $\frac{3}{2}^+$ should this systematic trend be continued. This, however, appears unlikely in view of the most probable spin assignment for its mirror nucleus Sc^{43} as $J^\pi = \frac{7}{2}^-$.[23]

Finally, energy level diagrams showing l_n values, angular momenta, and parities of the levels observed in Ti^{49}, Ti^{48}, Ti^{47}, Ti^{46}, and Ti^{45} in the present (p,d) investigation are shown in Fig. 16.

CONCLUSIONS

The present investigation has shown that for the Ti isotopes, many of the important features of the (p,d) reaction to low-lying levels are well explained under the assumption that neutrons and protons in excess of 20 are in a $(1f_{7/2})^n$ configuration, and that attempts to treat the spectra in terms of only neutron configuration (assuming the protons to be coupled to 0) are inadequate. The next step in the calculation is to add $2p$ admixture, actually only $2p_{3/2}$ as no evidence was found for $2p_{1/2}$ admixture. The fact that some even parity levels appear to go by a direct reaction process in both pickup and stripping reactions is of great interest. The conclusion reached here that this effect reflects admixture from the lower lying $2s_{1/2}$ and $1d_{3/2}$ shells follows logically from the experimental results, but one may wish to question whether the knowledge of the reaction mechanism is sufficient to allow such conclusions about initial and final states.

[23] T. Lindquist and A. C. G. Mitchell, Phys. Rev. **95**, 1535 (1954).

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Isomer Ratios from (α,xn) Reactions on Silver*

C. T. Bishop† and J. R. Huizenga
Argonne National Laboratory, Argonne, Illinois

AND

J. P. Hummel
University of Illinois, Urbana, Illinois

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Excitation functions have been measured for the $Ag^{107}(\alpha,n)In^{110,110m}$ and $Ag^{109}(\alpha,3n)In^{110,110m}$ reactions with isotopically separated targets. The excitation functions for the high-spin metastable state peak in both reactions at higher bombarding energies than the low-spin ground state. In the case of the $Ag^{107}(\alpha,n)$ reaction, the cross section for the formation of In^{110} peaks at a helium-ion energy of about 17 MeV and that for In^{110m} peaks at about 19.5 MeV. The isomer ratio, $\sigma_m/(\sigma_m+\sigma_g)$, determined for the Ag^{107} (α,n) reaction varies from 0.13 at a helium-ion energy of 10.8 MeV to 0.81 at a helium-ion energy of 22.0 MeV. In the $Ag^{109}(\alpha,3n)$ reaction, this ratio varies from 0.68 at a helium-ion energy of 27.6 MeV to 0.87 at a helium-ion energy of 38.7 MeV. The experimental cross sections are based on measured half-lives of 70.2 ± 1.4 min and 5.2 ± 0.2 h for In^{110} and In^{110m}, respectively. The isomer ratios were calculated theoretically for the above reactions and the effects of various parameters on the calculations were examined. The experimental isomer ratios for the $Ag^{107}(\alpha,n)$ reaction for bombarding energies below 18 MeV agree within experimental uncertainties with calculated results based on either a Fermi-gas model with a rigid moment of inertia ($\sigma^2=34.7t$) or a superconductor model. A superconductor model predicts only about a 20% reduction in the moment of inertia for In^{110} and such a small change could not be definitely established from the data. A marked increase in the experimental isomer ratios from the $Ag^{107}(\alpha,n)$ reaction is observed near the onset of the $(\alpha,2n)$ reaction. This increase is probably due to a fractionation of the intermediate spin distribution for energies slightly exceeding the threshold of a second reaction. This effect is suggested also in the $Ag^{109}(\alpha,3n)$ reaction by the small experimental isomer ratios at bombarding energies where the $(\alpha,2n)$ competition is sizeable. These results indicate that values of σ deduced from the isomer ratio technique are in error at energies where cross sections for a competing reaction are large.

I. INTRODUCTION

IN this investigation angular momentum effects in the $Ag^{107}(\alpha,n)$ and the $Ag^{109}(\alpha,3n)$ reactions at several helium-ion bombarding energies have been studied by comparing experimentally determined isomer ratios of $In^{110,110m}$ with theory. The isomer ratio is defined as

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† Present address: Villa Madonna College, Covington, Kentucky.